

Resonance in Spherical-Circular Microstrip Structures

Wai-Yip Tam, *Member, IEEE*, and Kwai-Man Luk, *Member, IEEE*

Abstract—The resonance problem of a circular microstrip disk mounted on a spherical surface is studied theoretically. The radiator is replaced by a surface current distribution. The effects of the dielectric substrate as well as the curvature effect are taken into account by the Green's function formulation in the spectral domain. A new vector Legendre series is defined. Cavity model current distribution is used as the current basis. Galerkin's procedure is employed to solve for the complex resonant frequencies. Some numerical results are given to illustrate the effects of curvature and dielectric substrate on the resonance of the microstrip patch.

I. INTRODUCTION

WHEN an antenna is mounted on a curved surface such as that of a missile, a satellite, or the roof of a car, it is advantageous if the element does not disturb the streamline of the body. Microstrip structures offer this capability because of their low profile and their conformity with a curved surface.

In the last decade, the resonance frequencies of microstrip patches placed on planar structure have been studied extensively [1]–[5]. However, the radiation characteristics of microstrip patch structures placed on curved surfaces have attracted little attention. Recently, papers have been published on evaluating the curvature effect of a body on the resonance frequencies of a microstrip patch mounted on an infinitely long cylindrical ground plane, e.g. [6] and [7].

In [6], a cavity model was used to calculate the resonant frequencies of a cylindrical-rectangular microstrip patch. In that analysis, both the fringing field and radiation were ignored. Thus, no information about the Q factor of the patch was obtained. In [7], the complex resonant frequencies of both cylindrical-rectangular and wraparound structures were analyzed using a full-wave approach.

In this paper, the complex resonant frequencies of a circular microstrip patch mounted on a spherical body are investigated using the spectral-domain method. In this method, the effects of the dielectric substrate and of the metallic sphere are incorporated by the rigorous Green's function formulation. A new vector Legendre series is defined. Galerkin's procedure is employed to solve for the complex resonant frequencies. The cavity model surface current functions are used as the basis functions. The resonant frequencies and Q factors of the TM_{11} mode are presented.

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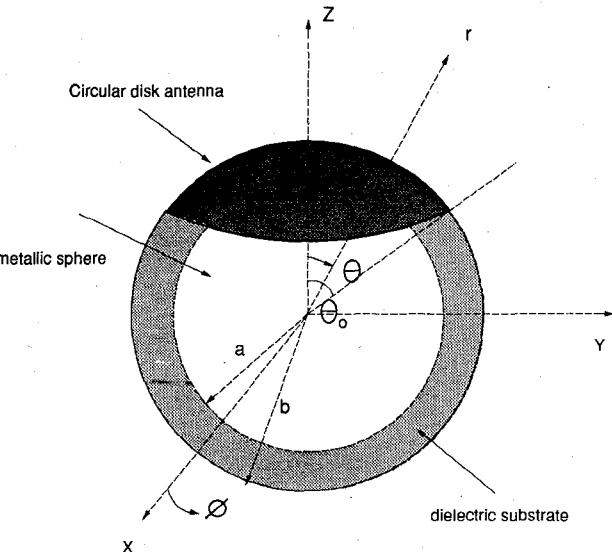


Fig. 1. Geometry of a circular disk on the surface of a sphere.

II. FORMULATION

The geometry of the microstrip structure mounted on a sphere is shown in Fig. 1. Although the following derivation is generally true for patches of any shape, only the circular disk will be analyzed, for brevity. In this figure, a metallic sphere of radius a is covered with a dielectric substrate of thickness h , permittivity ϵ_1 , and permeability μ_0 . The outer region ($r > b = a + h$) is free space with permittivity ϵ_0 and permeability μ_0 . A metallic patch is printed on the surface of the dielectric substrate. The metallic sphere and the patch are assumed to be perfect conductors. The thickness of the disk is negligible, as it is extremely small compared with the wavelength.

Based on the above assumptions, the radiator is replaced by an assumed surface current distribution \mathbf{J}_s at $r = b$. The fields are assumed to vary harmonically as $e^{j\omega t}$ and are suppressed.

In a source-free region, the field components can be calculated from the electric $A_r \hat{r}$ and magnetic $F_r \hat{r}$ potentials, which generate a transverse magnetic to r (TM) field and a transverse electric (TE) to r field respectively. These functions satisfy the scalar Helmholtz equation in spherical coordinates so that they can be expressed in terms of spherical harmonics [8].

Inside the dielectric ($a < r < b$), two spherical Bessel functions are needed to represent the solution since there are two boundaries to be matched. In addition, the associated Legendre function of the first kind is used to construct the

solution as the fields are finite at $\theta = 0$ and π . Hence, we write

$$A_{r1} = e^{jm\phi} \sum_{n=m}^{\infty} [A(n)\hat{H}_n^{(1)}(kr) + B(n)\hat{H}_n^{(2)}(kr)] P_n^m(\cos \theta) \quad (1a)$$

$$F_{r1} = e^{jm\phi} \sum_{n=m}^{\infty} [C(n)\hat{H}_n^{(1)}(kr) + D(n)\hat{H}_n^{(2)}(kr)] P_n^m(\cos \theta), \quad a < r < b \quad (1b)$$

$$k = \omega\sqrt{\mu_0\epsilon_1}.$$

Similarly, a spherical Hankel function of the second kind is used for $r > b$, as it represents an outward-traveling wave:

$$A_{r2} = e^{jm\phi} \sum_{n=m}^{\infty} E(n)\hat{H}_n^{(2)}(k_0r) P_n^m(\cos \theta) \quad (2a)$$

$$F_{r2} = e^{jm\phi} \sum_{n=m}^{\infty} F(n)\hat{H}_n^{(2)}(k_0r) P_n^m(\cos \theta), \quad r > b \quad (2b)$$

$$k_0 = \omega\sqrt{\mu_0\epsilon_0}$$

where the coefficients $A(n)$, $B(n)$, $C(n)$, $D(n)$, $E(n)$, and $F(n)$ are functions of the harmonic order n , $P_n^m(x)$ is the associated Legendre function of the first kind with order m and degree n , and $\hat{H}_n(x)$ is the spherical Hankel function (Schelkunoff type) of order n [8].

The transverse field components can be expressed in terms of these two potential functions [9]. For $r > b$,

$$E_{\theta 2} = e^{jm\phi} \sum_{n=m}^{\infty} \left(-\frac{jm}{r \sin \theta} F(n)\hat{H}_n^{(2)}(k_0r) P_n^m(\cos \theta) + \frac{k_0}{j\omega\epsilon_0 r} E(n)\hat{H}_n^{(2)\prime}(k_0r) \frac{\partial P_n^m(\cos \theta)}{\partial \theta} \right) \quad (3a)$$

$$E_{\phi 2} = e^{jm\phi} \sum_{n=m}^{\infty} \left(\frac{1}{r} F(n)\hat{H}_n^{(2)}(k_0r) \frac{\partial P_n^m(\cos \theta)}{\partial \theta} + \frac{mk_0}{\omega\epsilon_0 r \sin \theta} E(n)\hat{H}_n^{(2)\prime}(k_0r) P_n^m(\cos \theta) \right) \quad (3b)$$

$$H_{\theta 2} = e^{jm\phi} \sum_{n=m}^{\infty} \left(\frac{jm}{r \sin \theta} E(n)\hat{H}_n^{(2)}(k_0r) P_n^m(\cos \theta) + \frac{k_0}{j\omega\mu_0 r} F(n)\hat{H}_n^{(2)\prime}(k_0r) \frac{\partial P_n^m(\cos \theta)}{\partial \theta} \right) \quad (3c)$$

$$H_{\phi 2} = e^{jm\phi} \sum_{n=m}^{\infty} \left(-\frac{1}{r} E(n)\hat{H}_n^{(2)}(k_0r) \frac{\partial P_n^m(\cos \theta)}{\partial \theta} + \frac{mk_0}{\omega\mu_0 r \sin \theta} F(n)\hat{H}_n^{(2)\prime}(k_0r) P_n^m(\cos \theta) \right). \quad (3d)$$

For $a < r < b$, the set of equation is similar to (3), except that

$E(n)\hat{H}_n^{(2)}(k_0r)$ is replaced by

$$A(n)\hat{H}_n^{(1)}(kr) + B(n)\hat{H}_n^{(2)}(kr)$$

and

$F(n)\hat{H}_n^{(2)}(k_0r)$ is replaced by

$$C(n)\hat{H}_n^{(1)}(kr) + D(n)\hat{H}_n^{(2)}(kr).$$

To simplify the problem, the transverse fields are expressed in the form of vector Legendre series. The transform pair is defined in matrix form:

$$\mathbf{F}(\theta) = \sum_{n=m}^{\infty} \bar{\bar{L}}(n, m, \theta) \tilde{\mathbf{F}}(n) \quad (4a)$$

$$\tilde{\mathbf{F}}(n) = \frac{1}{S(n, m)} \int_0^{\pi} \bar{\bar{L}}(n, m, \theta) \mathbf{F}(\theta) \sin \theta d\theta \quad (4b)$$

where $\mathbf{F}(\theta)$ and $\tilde{\mathbf{F}}(n)$ are column matrices and $\bar{\bar{L}}(n, m, \theta)$ is a 2×2 matrix:

$$\mathbf{F}(\theta) = \begin{bmatrix} F_{\theta} \\ F_{\phi} \end{bmatrix}$$

$$\tilde{\mathbf{F}}(n) = \begin{bmatrix} \tilde{F}_{\theta} \\ \tilde{F}_{\phi} \end{bmatrix}$$

$$\bar{\bar{L}}(n, m, \theta) = \begin{bmatrix} \frac{\partial P_n^m(\cos \theta)}{\partial \theta} & \frac{-jmP_n^m(\cos \theta)}{\sin \theta} \\ \frac{jmP_n^m(\cos \theta)}{\sin \theta} & \frac{\partial P_n^m(\cos \theta)}{\partial \theta} \end{bmatrix}.$$

Also, in (4b),

$$S(n, m) = \frac{2n(n+1)(n+m)!}{(2n+1)(n-m)!}.$$

The spectral coefficient $\tilde{\mathbf{F}}(n)$ in (4b) can be evaluated from (4a) on the basis of the orthogonal property of the Legendre function [10]. Therefore, by suppressing the term $e^{jm\phi}$, we have

$$\tilde{\mathbf{E}}_1 = \begin{bmatrix} \frac{k}{j\omega\epsilon_1 r} [A(n)\hat{H}_n^{(1)\prime}(kr) + B(n)\hat{H}_n^{(2)\prime}(kr)] \\ \frac{1}{r} [C(n)\hat{H}_n^{(1)}(kr) + D(n)\hat{H}_n^{(2)}(kr)] \end{bmatrix} \quad (5a)$$

$$\tilde{\mathbf{H}}_1 = \begin{bmatrix} \frac{k}{j\omega\mu_0 r} [C(n)\hat{H}_n^{(1)\prime}(kr) + D(n)\hat{H}_n^{(2)\prime}(kr)] \\ -\frac{1}{r} [A(n)\hat{H}_n^{(1)}(kr) + B(n)\hat{H}_n^{(2)}(kr)] \end{bmatrix} \quad (5b)$$

$$\tilde{\mathbf{E}}_2 = \begin{bmatrix} \frac{k_0}{j\omega\epsilon_0 r} E(n)\hat{H}_n^{(2)\prime}(k_0r) \\ \frac{1}{r} F(n)\hat{H}_n^{(2)}(k_0r) \end{bmatrix} \quad (5c)$$

$$\tilde{\mathbf{H}}_2 = \begin{bmatrix} \frac{k_0}{j\omega\mu_0 r} F(n)\hat{H}_n^{(2)\prime}(k_0r) \\ -\frac{1}{r} E(n)\hat{H}_n^{(2)}(k_0r) \end{bmatrix} \quad (5d)$$

The unknown coefficients $A(n)$ through $F(n)$ have to be solved together with the following boundary conditions:

$$\hat{\mathbf{r}} \times \vec{\mathbf{E}}_1 = 0 \quad \text{on the surface } r = a \quad (6a)$$

$$\hat{\mathbf{r}} \times (\vec{\mathbf{E}}_2 - \vec{\mathbf{E}}_1) = 0 \quad \text{on the surface } r = b \quad (6b)$$

$$\hat{\mathbf{r}} \times (\vec{\mathbf{H}}_2 - \vec{\mathbf{H}}_1) = \vec{\mathbf{J}}_s \quad \text{on the surface } r = b. \quad (6c)$$

In the spectral domain, the corresponding boundary conditions are

$$\tilde{E}_1(n) = 0 \quad (7a)$$

$$\tilde{E}_1(n) = \tilde{E}_2(n) \quad (7b)$$

$$\tilde{J}_s(n) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} [\tilde{H}_2(n) - \tilde{H}_1(n)]. \quad (7c)$$

From (7a), the relation between inward and outward waves inside the dielectric substrate can be determined. In this equation, the tangential electric field vanishes on the surface of the metallic sphere, and we have

$$B(n) = -\alpha_{\text{TM}} A(n) \quad (8a)$$

$$D(n) = -\alpha_{\text{TE}} C(n) \quad (8b)$$

where

$$\alpha_{\text{TM}} = \frac{\hat{H}_n^{(1)'}(ka)}{\hat{H}_n^{(2)'}(ka)} \quad (9a)$$

$$\alpha_{\text{TE}} = \frac{\hat{H}_n^{(1)}(ka)}{\hat{H}_n^{(2)}(ka)} \quad (9b)$$

By subjecting boundary conditions (7b) and (7c) to the spectral field components, we have

$$\tilde{J}_s = \bar{\bar{Y}} \tilde{E} \quad (10)$$

where $\bar{\bar{Y}}$ is a diagonal matrix

$$\bar{\bar{Y}} = \begin{bmatrix} jY_0 \frac{H_1(b)}{H_1'(b)} - jY_1 \frac{H_3(b)}{H_3'(b)} & 0 \\ 0 & jY_0 \frac{H_1'(b)}{H_1(b)} - jY_1 \frac{H_2(b)}{H_2'(b)} \end{bmatrix} \quad (11)$$

where

$$Y_i = \sqrt{\frac{\epsilon_i}{\mu_0}} \quad (12)$$

$$H_1(r) = \hat{H}_n^{(2)}(k_0 r) \quad (13a)$$

$$H_2(r) = \{\hat{H}_n^{(1)}(kr) - \alpha_{\text{TE}} \hat{H}_n^{(2)}(kr)\} \quad (13b)$$

$$H_3(r) = \{\hat{H}_n^{(1)}(kr) - \alpha_{\text{TM}} \hat{H}_n^{(2)}(kr)\}. \quad (13c)$$

Equation (10) may be written as

$$\tilde{E} = \bar{\bar{Z}} \tilde{J}_s \quad (14)$$

where $\bar{\bar{Z}} = \bar{\bar{Y}}^{-1}$.

Next, the surface current is expanded by a set of basis functions:

$$\tilde{J}_s = \sum_j^N a_j \tilde{J}_{sj}.$$

Hence

$$\tilde{E} = \sum_j^N a_j \bar{\bar{Z}} \tilde{J}_{sj} \quad (15)$$

Applying Galerkin's procedure, the same set of basis functions are taken as the weighting functions, and the inner

product is defined as follows:

$$\langle A, B \rangle = \sum_{n=m}^{\infty} A^+(n) B(n) S(n, m) \quad (16)$$

where the superscript + denotes the complex conjugate transpose.

When (14) is weighted by each weighting function, we have

$$\sum_{n=m}^{\infty} \tilde{J}_{si}^+ S(n, m) \tilde{E} = \sum_{n=m}^{\infty} \tilde{J}_{si}^+ S(n, m) \left[\sum_j^N a_j \bar{\bar{Z}} \tilde{J}_{sj} \right]. \quad (17)$$

The right-hand-side of (17) becomes

$$\int_0^{\pi} J_{si}^+(\theta) E(\theta) \sin \theta d\theta = 0$$

by Perserval's theorem.

The above integral vanishes as the surface current and the electric field are complementary to each other on the surface $r = b$. Therefore, (17) becomes

$$\sum_j^N a_j \left[\sum_{n=m}^{\infty} \tilde{J}_{si}^+ S(n, m) \bar{\bar{Z}} \tilde{J}_{sj} \right] = 0. \quad (18)$$

In matrix form,

$$PA = 0 \quad (19)$$

where

$$A = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_N \end{bmatrix} \quad (20)$$

and

$$P_{ij} = \sum_{n=m}^{\infty} \tilde{J}_{si}^+ S(n, m) \bar{\bar{Z}} \tilde{J}_{sj}. \quad (21)$$

For the existence of a nontrivial solution to the homogeneous equation, (19), the determinant of P should vanish, i.e.,

$$\text{Det}(P) = 0. \quad (22)$$

In general, the solution of this equation, denoted by ω , of this equation is a complex number. The imaginary part of ω , denoted by $\mathcal{J}(\omega)$, accounts for the radiation loss and gives the quality factor, Q , of the structure. The real part of the solution, denoted by $\mathcal{R}(\omega)$, is the resonant frequency, ω_r , of the structure. In mathematical form, we have

$$\omega_r = \mathcal{R}(\omega) \quad (23)$$

$$Q = \frac{\mathcal{R}(\omega)}{2\mathcal{J}(\omega)}. \quad (24)$$

The surface current distribution in (15) is approximated by a set of basis functions. Theoretically, an infinite number of basis functions should be used to give an exact expansion of the surface current. However, only a few basis functions are adequate to provide a practical solution if the basis functions are chosen appropriately. Furthermore, as the current basis functions should be transformed in evaluating the complex resonant frequency, it is convenient to choose expressions that are transformable into closed forms. In the analysis, the cavity model theory [1] is employed to derive the basis

functions for the surface current distribution. To this end, the surface currents on the circular disk operated at the TM_{mv} mode and TE_{mv} mode are obtained as follows. For TM_{mv} modes,

$$J_s^{\text{TM}_{mv}}(\theta) = \begin{cases} \left[\begin{array}{c} \frac{\partial P_v^m(\cos \theta)}{\partial \theta} \\ \frac{jm}{\sin \theta} P_v^m(\cos \theta) \\ 0 \end{array} \right], & \theta < \theta_0 \\ \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right], & \theta > \theta_0 \end{cases} \quad (25a)$$

with

$$P_v^{m'}(\cos \theta_0) = 0. \quad (25b)$$

For TE_{mv} modes,

$$J_s^{\text{TE}_{mv}}(\theta) = \begin{cases} \left[\begin{array}{c} \frac{jm}{\sin \theta} P_v^m(\cos \theta) \\ -\frac{\partial P_v^m(\cos \theta)}{\partial \theta} \\ 0 \end{array} \right], & \theta < \theta_0 \\ \left[\begin{array}{c} 0 \\ 0 \\ 0 \end{array} \right], & \theta > \theta_0 \end{cases} \quad (26a)$$

with

$$P_v^m(\cos \theta_0) = 0. \quad (26b)$$

From the definition of the vector Legendre series, (4), the amplitude coefficients of the TM_{mv} and TE_{mv} mode surface current distributions are given by

$$J_s^{\text{TM}_{mv}} = \frac{1}{S(n, m)} \left[\begin{array}{c} (1 - \cos^2 \theta_0) \frac{v(v+1)}{n(n+1) - v(v+1)} P_v^m(\cos \theta_0) P_n^{m'}(\cos \theta_0) \\ jm P_n^m(\cos \theta_0) P_v^m(\cos \theta_0) \end{array} \right] \quad (27)$$

$$J_s^{\text{TE}_{mv}} = \frac{1}{S(n, m)} \left[\begin{array}{c} 0 \\ (1 - \cos^2 \theta_0) \frac{n(n+1)}{n(n+1) - v(v+1)} P_n^m(\cos \theta_0) P_v^{m'}(\cos \theta_0) \end{array} \right]. \quad (28)$$

III. NUMERICAL RESULTS

In this section, the effects of the dielectric substrate and of the size of the sphere on the resonant frequencies and Q factors of a circular disk patch are illustrated with some practical examples.

In parts (a) and (b) of Fig. 2, the resonant frequencies and Q factors of the TM_{11} mode are displayed as a function of the dielectric substrate thickness. The curves correspond to the different numbers of surface current modes used. It can be observed that the Q factor converges much faster than the resonant frequency.

Parts (a) and (b) of Fig. 3 show the resonant frequencies and Q factors for different sizes of the metallic sphere with a

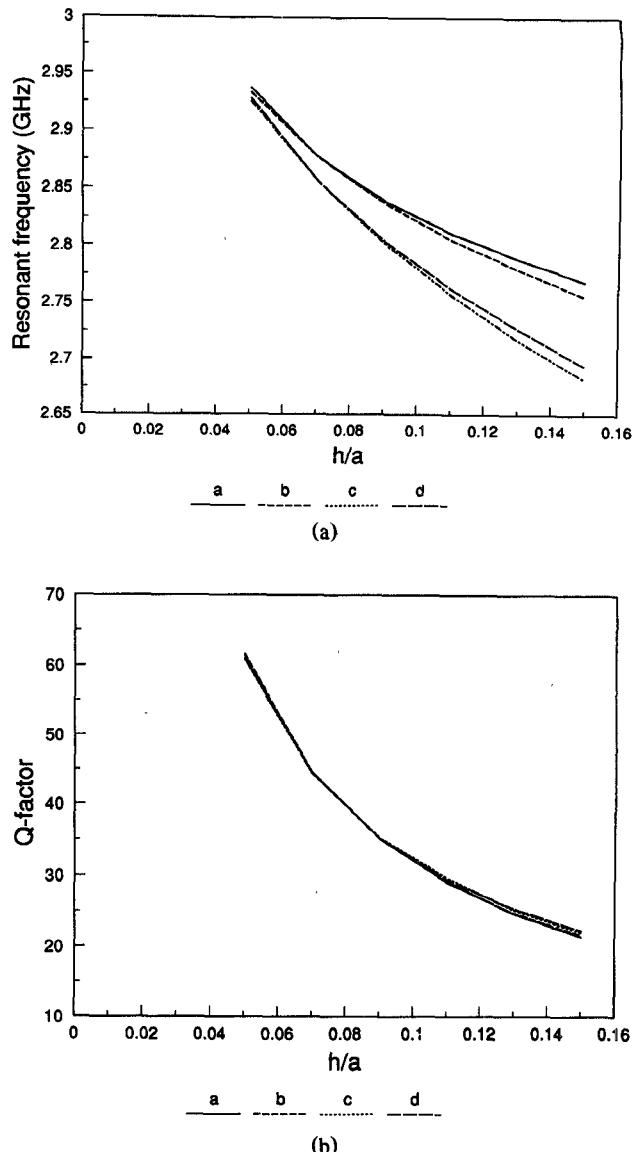
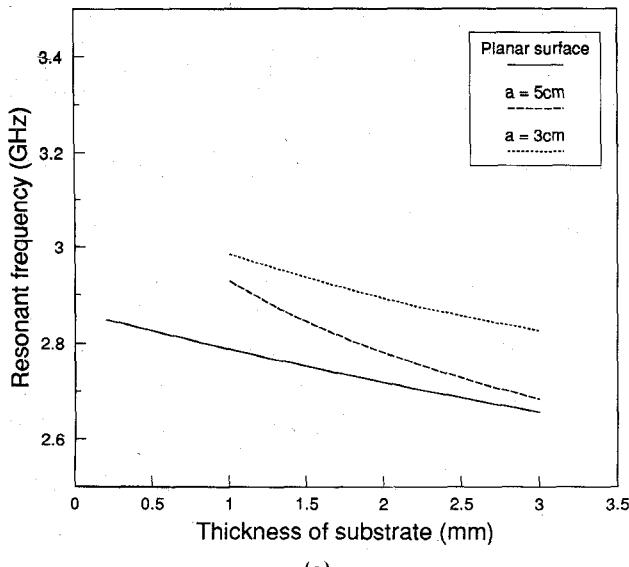


Fig. 2. (a) Resonant frequency with different numbers of basis current: $\epsilon_r = 2.32$, $b\theta_0 = 2$ cm, $a = 5$ cm. Curve a: TM_{11} , Curve b: TM_{11} and TE_{11} , Curve c: TM_{11} and TM_{12} , Curve d: TM_{11} , TM_{12} , and TE_{11} . (b) Q factor with different numbers of basis current: $\epsilon_r = 2.32$, $b\theta_0 = 2$ cm, $a = 5$ cm. Curve a: TM_{11} , Curve b: TM_{11} and TE_{11} , Curve c: TM_{11} and TM_{12} , Curve d: TM_{11} , TM_{12} , and TE_{11} .

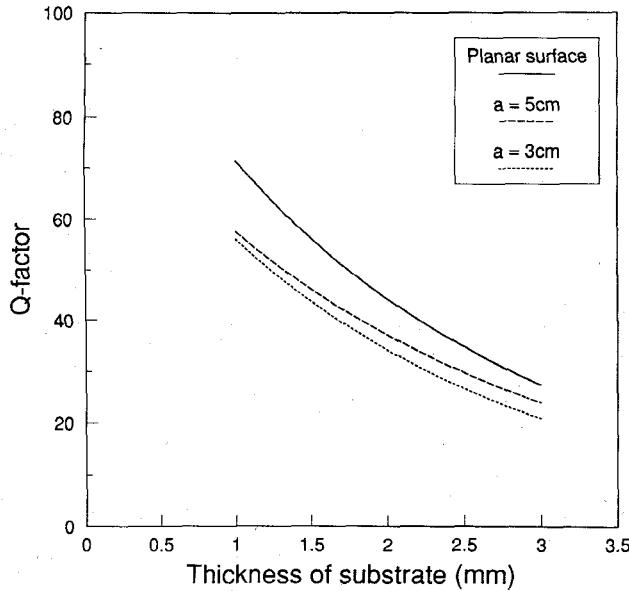
substrate having a dielectric constant $\epsilon_r = 2.32$. From these graphs, it is seen that the curved surface reduces the effective radius of the patch and increases the radiation loss of the structure. The curves for the planar case are obtained by the Hankel domain analysis [3].

IV. CONCLUSION

The resonant frequency and the quality factor of a circular disk mounted on a spherical body, operated at the TM_{11} mode, have been studied rigorously using the spectral-domain method. Numerical results show that the effect of the curved surface on the resonant frequency and quality factor may be significant. In general, when the radius of the sphere is decreased, the effective radius of the patch will be reduced and the radiation loss is increased.



(a)



(b)

Fig. 3. (a) Resonant frequency with different sphere radii: $\epsilon_r = 2.32$, $b\theta_0 = 2$ cm. TM₁₁, TM₁₂, and TE₁₁ modes are used. (b) Q factor with different sphere radii. $\epsilon_r = 2.32$, $b\theta_0 = 2$ cm. TM₁₁, TM₁₂, and TE₁₁ modes are used.

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